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# A Study of Line of Bearing Errors

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Many systems use Line-of-Bearing (LOB) information to locate distant objects. The errors in location due to noisy LOB measurements are the focus of this report. A central assumption for these systems is that the LOB location errors are adequately modeled by bivariate Gaussian distributions. These systems typically use weighted averages to estimate a target's location.

A discussion in Thompson and Durfee (1992) illustrates that these location errors are not necessarily Gaussian. In some situations, reasonable errors lead to calculation of implausible target locations. In addition, it was demonstrated that the location distribution is skewed in the direction of increasing range; thus, bias is a problem.

This report investigates the properties of the errors associated with LOB locations. Guidelines are suggested for various assumptions about LOB location errors. The concept of the stability ratio is introduced and used to determine the possibility of encountering implausible LOB locations; as a means of predicting the existence and magnitude of estimator bias; to determine the applicability of using a bivariate normal distribution for modeling LOB errors; and for determining the utility of closed-form models to predict the covariance of LOB locations.

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## 1. INTRODUCTION

Many systems use Line-of-Bearing (LOB) information to locate distant objects. The errors in location due to noisy LOB measurements are the focus of this report. A central assumption for these systems is that the LOB location errors are adequately modeled by bivariate Gaussian distributions. These systems typically use weighted averages to estimate a target's location. Weighted averages for bivariate data are discussed in Alexander (1980) and Thompson (1991a). Recursive estimators update the estimate by a gradient based on the current observation. The Kalman filter is a recursive estimator that uses the covariance of an observation to determine the influence of that observation on the current estimate.

A discussion in Thompson and Durfee (1992) illustrates that these location errors are not necessarily Gaussian. In some situations, reasonable errors lead to calculations of implausible target locations. In addition, it was demonstrated that the location distribution is skewed in the direction of increasing range; thus, bias is a problem.

This report investigates the properties of the errors associated with LOB locations. Guidelines are suggested for various assumptions about LOB location errors. The concept of the stability ratio is introduced and used to determine the possibility of encountering implausible LOB locations, as a means of predicting the existence and magnitude of estimator bias, to determine the applicability of using a bivariate normal distribution for modeling LOB errors, and for determining the utility of closed-form models to predict the covariance of LOB locations.

## 2. BACKGROUND

A LOB measurement is a direction from a point to a target. An estimate of a target's location can be found by combining two separated LOB measurements. This estimate is usually called a "fix" or a "cut." The segment between the two LOB sensors is referred to as the "baseline," and the distance to the target from the center of the baseline is the "range." The system boresight is the portion of the perpendicular bisector of the baseline in the halfplane into which the sensors are directed. The off-boresight angle is the angle between the boresight and the direction to the target. These relationships are shown in Figure 1.

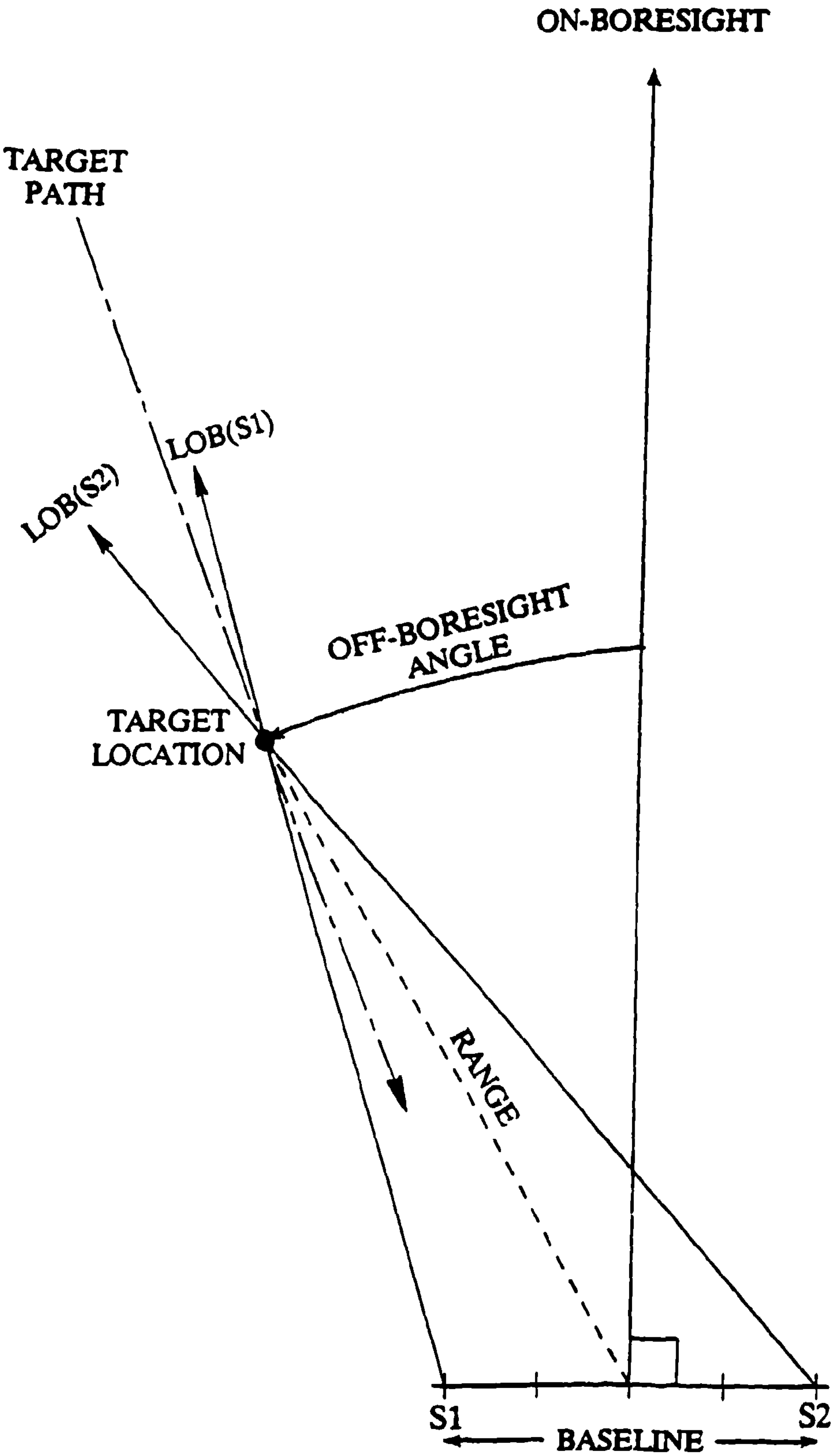


Figure 1. Basic definitions of LOB system geometry.

The geometric or physical relationship between the sensors and the target needs to be mentioned when discussing system performance. This can be addressed by an assessment of system performance at a set of locations or through a measure of the situational geometry. One measure of this is the range-to-baseline ratio. This descriptor is particularly useful, as its meaning is clear and easy to visualize. A more precise measure of the physical relationship is the angle formed by the rays connecting the target to each sensor. For targets at ranges in excess of one baseline, the angle descriptor (which decreases) more accurately describes the off-boresight case than the range-to-baseline ratio (which does not decrease). For the off-boresight situation, the effective baseline is found by multiplying the baseline by $\cos(\beta)$ where $\beta$ is the off-boresight angle. At longer ranges for a given off-boresight angle, these descriptors become equivalent as the tangent of the angle approaches the radial measure of the angle.

When the angle formed by the line segments connecting the sensors to the target is small, measurement errors can have a drastic effect on the estimate of the target location. Figure 2 shows the geometry for a simple two-dimensional LOB system. In this figure, S1 and S2 represent two LOB sensors separated by 2 units. These sensors are viewing a target at a distance of 4 units giving a range-to-baseline ratio of 2:1. The true LOBs from S1 and S2 intersect at the "true target location." On both sides of the bearing lines from S1 and S2 are drawn rays (dashed lines) which bracket hypothetical angular excursions between $\pm$ 3.0°. The area formed by the intersection of the 4 angular excursion rays from S1 and S2 provide an example of the extreme area in which the target might be observed (the hatched area in Figure 2). It should be noted that the range axis of the hatched area is significantly greater than the cross-range axis, and that the range axis is greater than 2 units long. A target located at 4 units might appear to be anywhere from 3 to more than 5 units away.

Figure 3 shows the result of relocating the sensors from S1 to S3 and from S2 to S4, giving a sensor separation of 1 unit. This results in a range-to-baseline ratio of 4:1. In this figure, the "target location area" from S1 and S2 (2-unit baseline separation) of Figure 2 is displayed as the darker cross-hatched area while the "target location area" for S3 and S4 (1-unit baseline separation) is displayed as the hatched area (which includes the cross-hatched area). It will be noted in Figure 3 that halving the sensor separation has very little affect on the the cross-range axis. However, the reduced sensor separation has caused a large increase in the length of the range axis. In this case, the extreme spread of the range excursions is now more than 5 units long and the target, at a true range of 4, which might appear to be from 2.5 to 7.7 units away. This indicates that the actual distribution is not symmetric about the mean and is skewed in range. For this skewed distribution, the mean is a biased estimator; it will overestimate the range to the target. This problem is discussed in a later section.

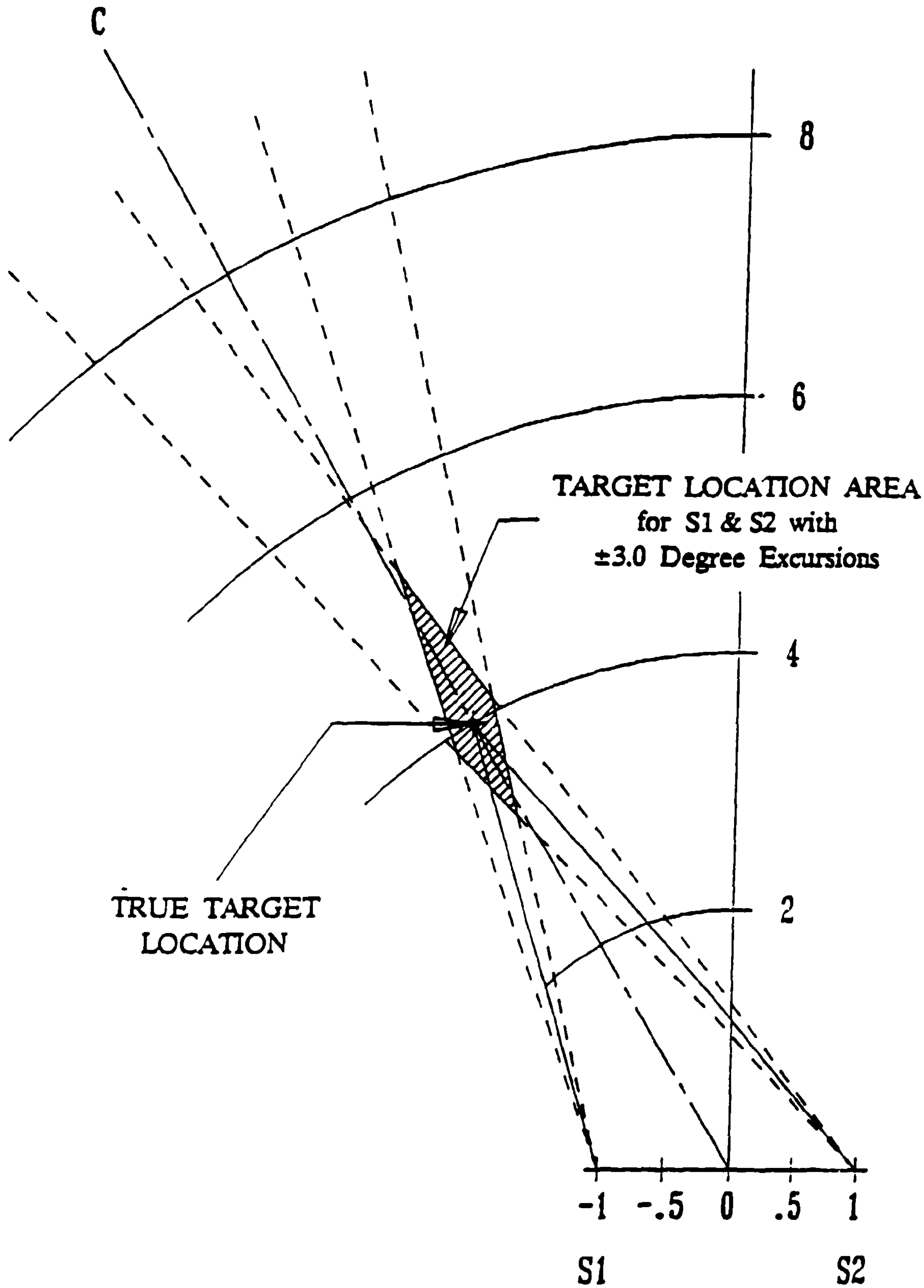


Figure 2. Two-dimensional LOB system geometry with 2:1 range-to-baseline ratio.

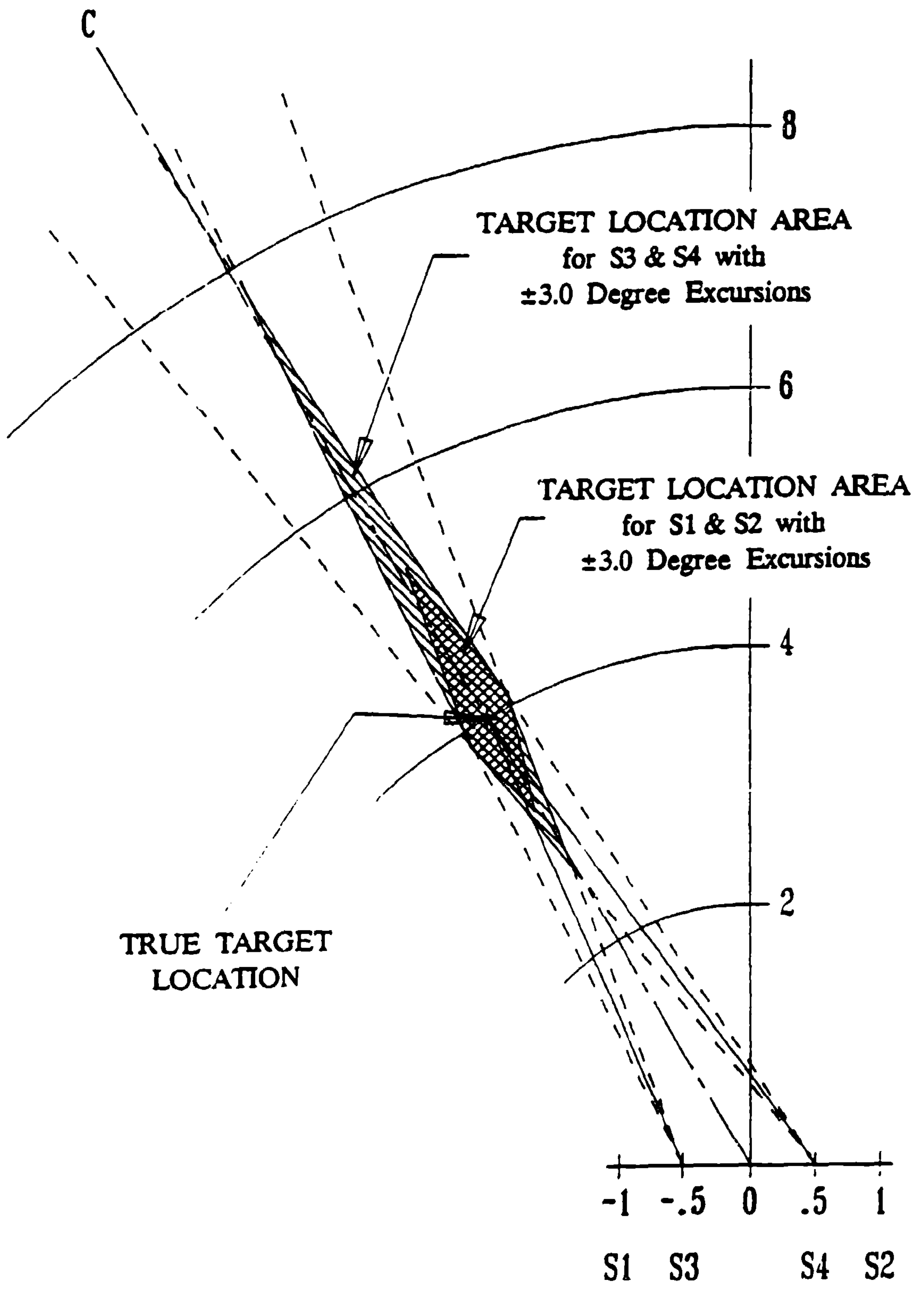


Figure 3. Two-dimensional LOB system geometry with 4:1 range-to-baseline ratio.

Further examples of the effects of LOB system geometry on range estimates for various angular excursions are plotted in the next four figures where graphs have been used rather than drawings so that larger range-to-baseline ratios can be presented. To standardize across many situations, the range is measured using the length of the baseline as the unit of measurement; thus, a target's range might be 4 baselines or 4 units. The effects of changing range-to-baseline ratio on extreme range excursions are plotted in Figures 4 and 5 for small angular excursions of 0.05°, 0.10°, and 0.50°. In these figures, negative extreme range excursion numbers indicate an observed target location nearer than the true location, and positive extreme range excursion numbers indicate an observed target location further than the true location.

Figure 4 shows an on-boresight 0° attack azimuth. Here, for example, LOB angular excursions of ± 0.5° could cause a target at a true range of 16 baselines to appear to be anywhere from 3.4 baselines closer to 6.1 baselines further than its actual range. In general, as the range-to-baseline ratio increases or the angular excursions increase, the extreme range excursions about the true target location get larger.

Figure 5 shows an off-boresight 40° attack azimuth. Here, for the true target location of 16 baselines mentioned above, the target might appear anywhere from 4.3 baselines closer to 9.2 baselines further than the actual range. Comparing these results with those from Figure 4 it will be noted that the off-boresight condition increases the extreme range excursions.

In some cases, the estimate of target location will be on the opposite side of the actual target or behind the baseline. This can be seen by considering the two LOBs to a distant target and visualizing a rotation to the right of the right line (rotate the true LOB of S2 clockwise in Figure 6) and likewise, a leftward rotation of the left line (in Figure 6 rotate the true LOB from S1 in a counterclockwise direction). Increasing the angular excursions, decreasing the baseline separation of the sensors, or increasing the range-to-target distance can exacerbate conditions in which the measured target location can appear to be behind the baseline. These effects can be seen in Figures 7 and 8 where the extreme range excursions are plotted as functions of range-to-target for angular excursions of 1°, 2°, and 3°.

In Figure 7, note that at a range of about 9.5 baselines for an angular excursion of 3° (14.5 baselines for angular excursion of 2°), the extreme range excursions go from a large positive value to a large negative value. This indicates that the two LOB sensor's bearing lines are crossing behind the baseline rather than in front of the baseline, resulting in a target that appears to be behind the baseline.

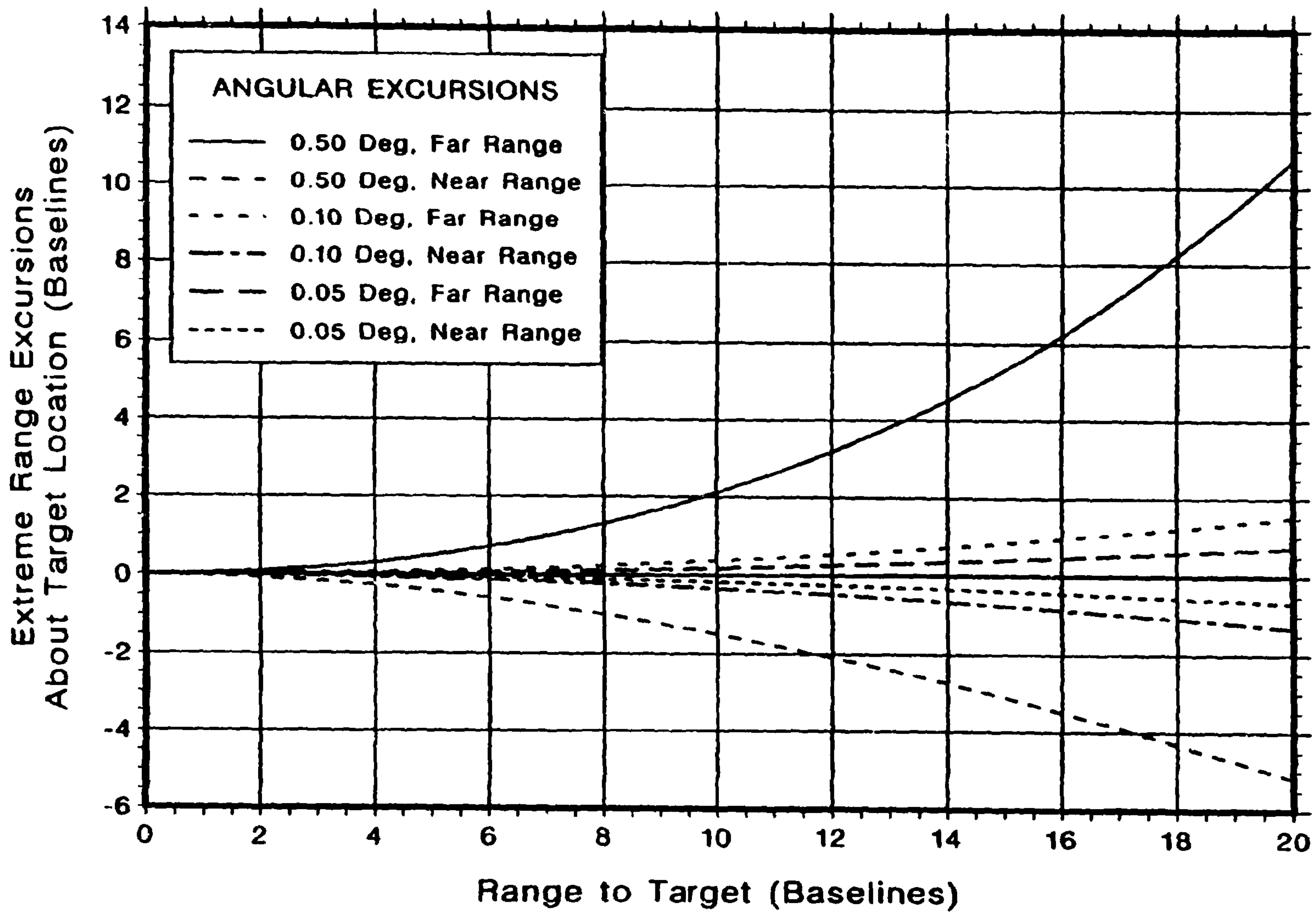


Figure 4. Geometric range excursions for 0.05° to 0.50° LOB systems, target on boresight.

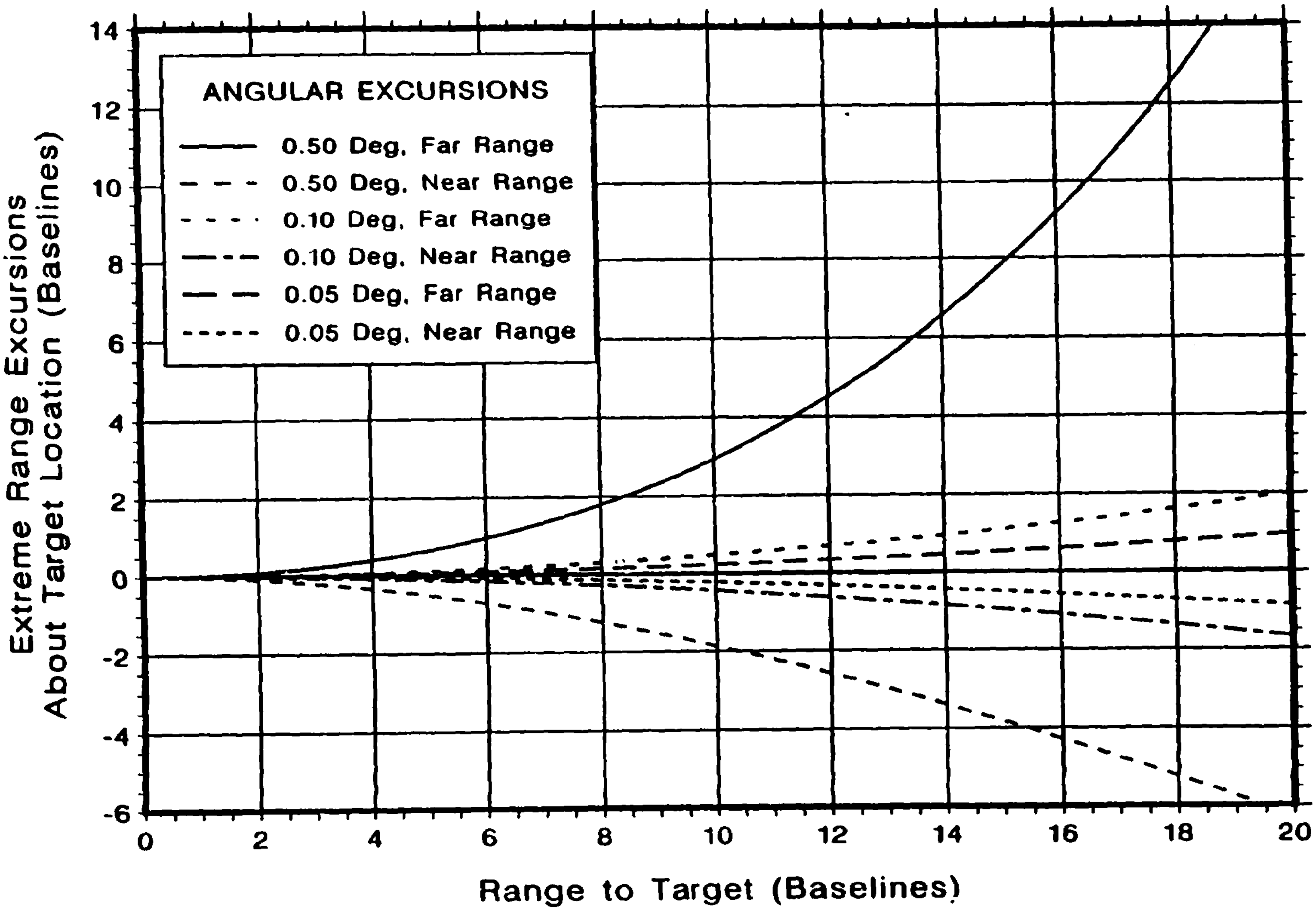


Figure 5. Geometric range excursions for 0.05° to 0.50° LOB systems, target 40° off-boresight.

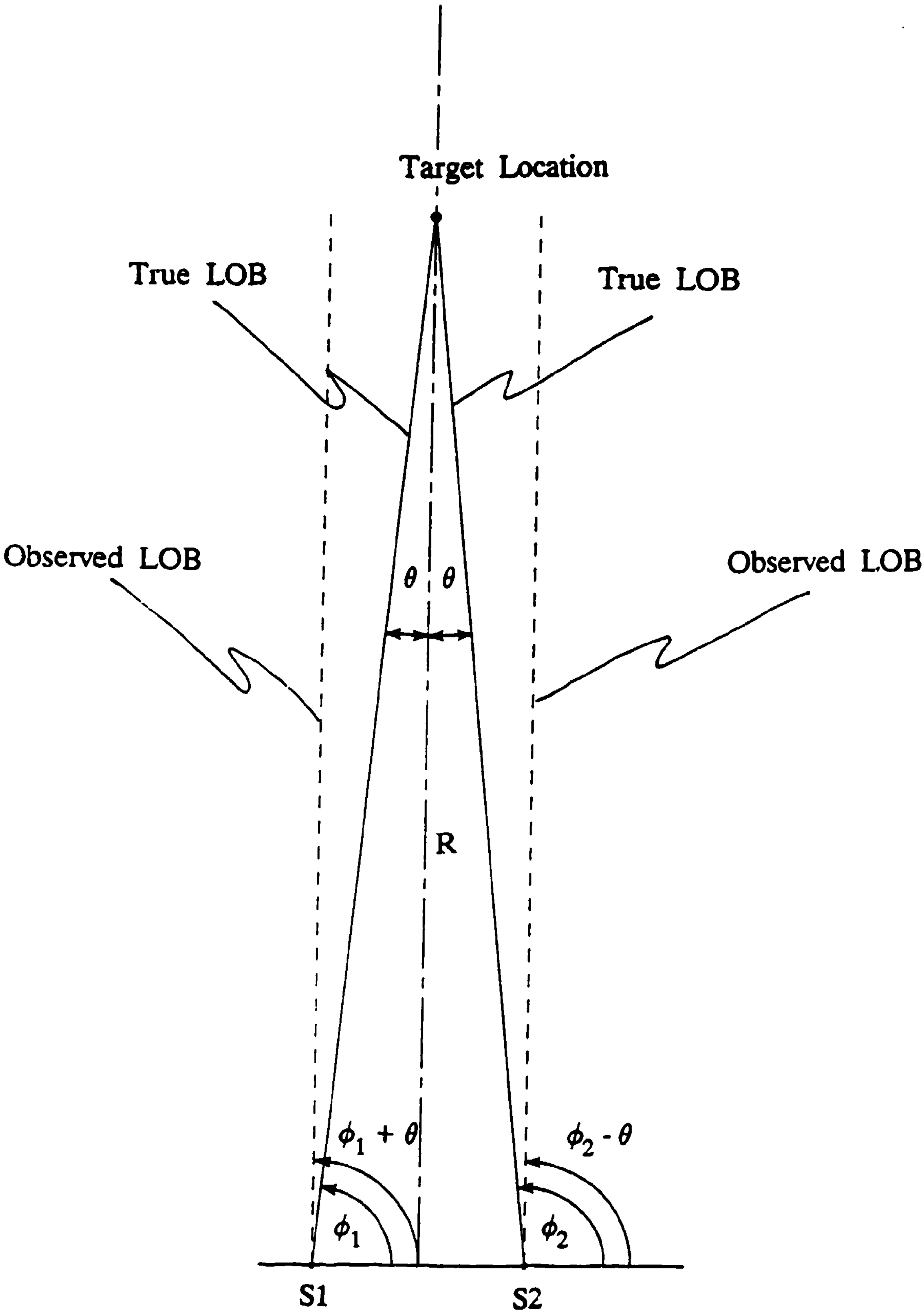


Figure 6. LOB systems geometry, target on-boresight.

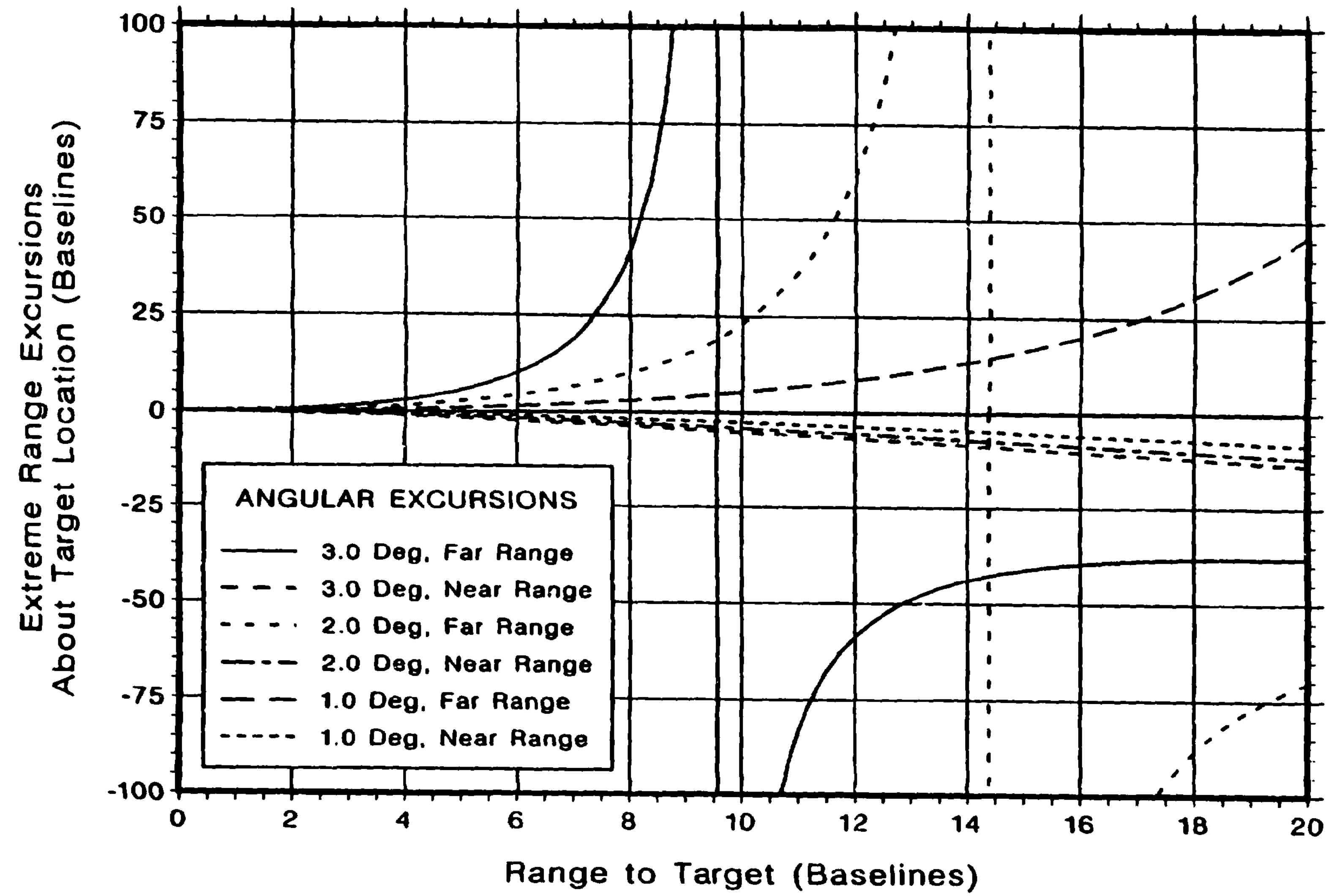


Figure 7. Geometric range excursions for 1.0° to 3.0° LOB systems, target on-boresight.

Figure 8 is the plot for the 40° off-boresight attack azimuth. Here the target appears behind the baseline at about 7.4 and 10.9 baselines for angular excursions of 3° and 2°, respectively. Comparing Figure 8 with Figure 7 indicates that off-boresight attacks can cause the target to appear behind the baseline at shorter ranges than the on-boresight attacks.

One can see that small changes in the measurement errors can lead to implausible location estimates. Systems processing LOB information should be designed to take into account these cases. This type of data association problem is a screening problem and is usually solved by gating or eliminating unlikely observations. Gating is usually based on physical knowledge or statistical hypothesis testing. When considering a LOB system for a mission, the possible geometries should be investigated to see if the mission is feasible.

When the two LOBs are parallel there is no solution. The measure of the set that is more extreme than the parallel line set is the probability that a target will be reported as being behind the sensors. The measure of the set in a small neighborhood of the parallel line situation would give the probability of extreme values. These extreme values could easily dominate estimates of the variance and the mean.

For most studies, closed-form error models are used to model system errors. One such model proposed by Dr. Charles Alexander is discussed in Appendix A of Thompson and Durfee (1992). This model will be referred to as LOBCA and will be one of the closed-form models used within this report. LOBCA develops a covariance model based on a trigonometric argument. Another closed-form model based on the ideas outlined in Thompson (1991b) is also used. This model, based on a linear algebra approach, is easily extended to predict three-dimensional covariances. One assumption of the closed-form models is that nonlinearity is insignificant in the area of interest. This assumption allows higher ordered partial derivatives to be ignored when considering the effects of perturbations or errors on the true situation. When a Kalman filter or weighted least-squares estimate is used, the covariances to associate with observations are usually computed from closed-form models.

## 3. IMPLAUSIBLE LOCATIONS

For an LOB system to perform well, the situations that result in parallel lines or estimates behind the sensors need to be minimized. As these situations become more prevalent, the variance of the estimate

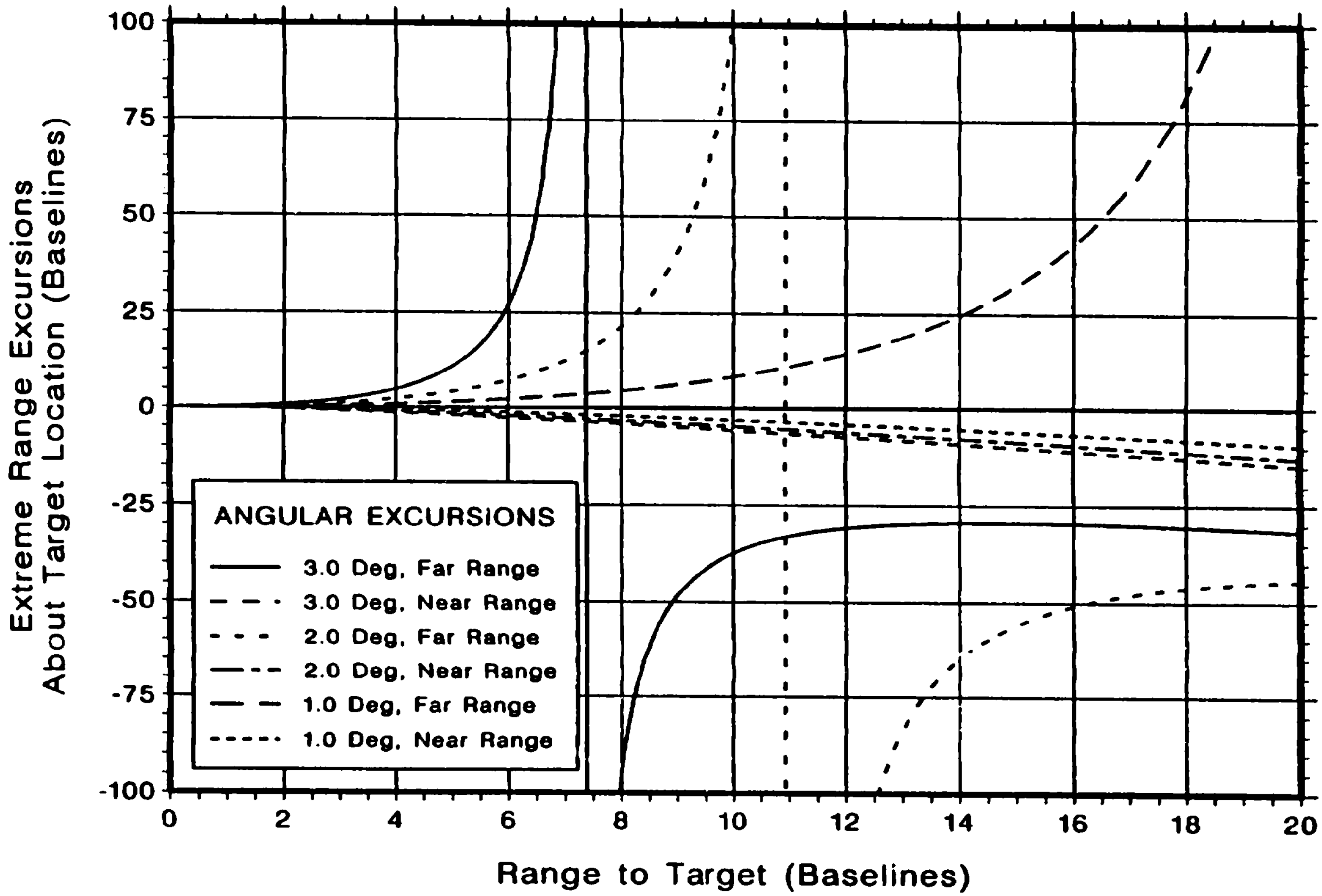


Figure 8. Geometric range excursions for 1.0° to 3.0° LOB systems, target 40° off-boresight.

will increase. In order to establish guidelines for recognizing poor system geometries, a simulation to generate LOB estimates, the mean of the estimates, and the variance of the estimates was made. The data generated by this simulation were used to develop guidelines and test a variety of assumptions. A guidance for minimizing implausible locations can be based on the geometry for the parallel line situation.

Figure 6 shows the LOB from sensor 1 to the target. The angle $\theta$ is the angle between the true LOB and the range line segment. If the observation error caused a counterclockwise rotation of the true LOB for sensor 1 (located at S1) through $\theta$, then it will be parallel to the range line segment. If a similar event took place for the true LOB from S2, then there would be no solution. For each value of $\phi_2$ (the LOB from sensor 2) if $\phi_1$ is greater than or equal to $\phi_2$, then the observed location will be an implausible location. The probability of this is

$$\int_0^{360} \int_{\phi_2}^{360} f(\phi_2)\, f(\phi_1)\, d\phi_1\, d\phi_2 \quad , \tag{1}$$

where $\phi_1$ and $\phi_2$ are the angles for the LOB from the two sensors, and $f(\phi_1)$, $f(\phi_2)$ are the probability density functions for the observed LOB from S1 and S2, respectively. Notice that both $f(\phi_1)$ and $f(\phi_2)$ are dependent on the geometry and the angular error. From Figure 6 it can be seen that the possibility of an observed LOB being greater than $\theta$ depends on the standard deviation of the measurement errors. If the target is on the perpendicular bisector of the baseline, the angle $\theta$ is formed between each LOB and the bisector. Then, for errors to cause the LOBs to be parallel for one LOB fixed, the second must be $2\theta$ more than the first. For this case, with normal errors of equal variances, the probability of errors that cause an implausible location is

$$\int_{-\infty}^{\infty} \int_{y+2\theta}^{\infty} (2\pi\sigma^2)^{-1} e^{-\frac{x^2+y^2}{2\sigma^2}}\, dx\, dy \ . \tag{2}$$

In the above equation, X and Y replace $\phi_1$ and $\phi_2$. In Equation 2, $\sigma$ is the standard deviation of the LOB error. The region described by Equation 2 is the probability measure of the subset of the circular normal distribution that is above a 45° line passing through the point $(0,\ 2\theta/\sigma)$. Figure 9 shows this region. Since rotations do not affect the measure of a set for a circular normal distribution, consider the effect of a −45° rotation. This rotation reduces the problem to finding the measure of the set that has a y value greater than $(\sqrt{2}/2)\ 2\theta/\sigma$. This is simply the probability that a standard normal random variable exceeds $\sqrt{2}\ \theta/\sigma$. There are several good approximations to this in addition to the standard normal

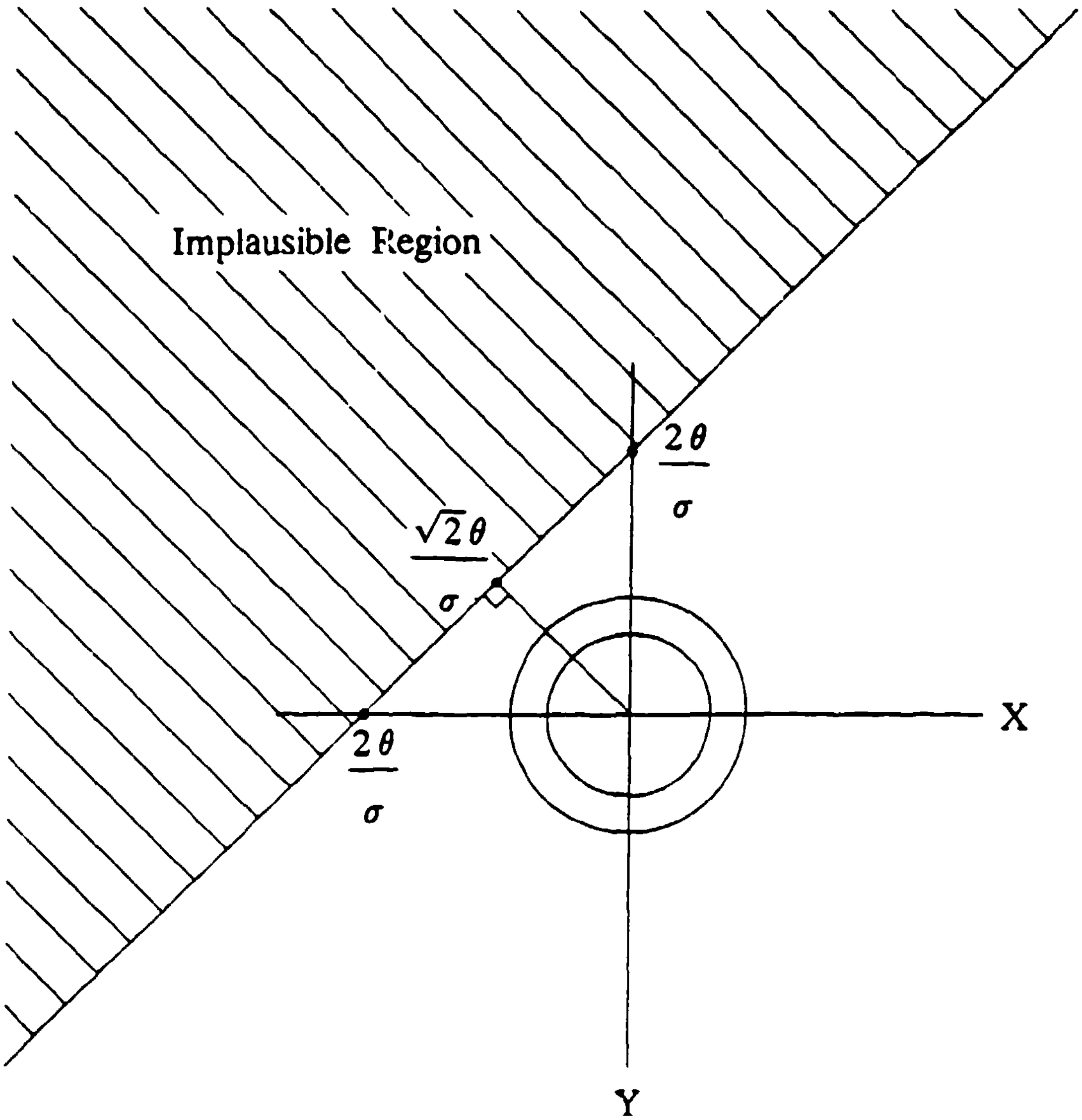


Figure 9. Distances to implausible region for circular normal LOB errors.

probability tables containing evaluations of this function. To find the probability of an implausible location, look up $p(z > \sqrt{2}\ \theta/\sigma)$, where z is a standard normal variable. Note that this gives the likelihood of getting a location estimate at infinity or behind the sensors. Locations approaching infinity will be included to the extent that the boundary value is reduced.

This probabilistic approach can be related to the geometry shown in Figure 6. Consider that $\tan(\theta) = 0.5/R$, where R is the range to the target in baselines. To find an acceptable performance range for the system, the risk of an implausible location can be set a priori; thus, $\sqrt{2}\ \theta/\sigma$ must be large enough so the probability of a standard normal random variable exceeding it is acceptably small. For example, if $\theta$ is chosen to be $\sigma$, the probability of a bad value is found to be 0.000011. Using the following formula, the safe range can be calculated for a given system.

$$R_{safe} = 0.5(\tan(3\sigma))^{-1} \qquad (3)$$

To find the maximum safe angular error for a system locating targets at a given range, use Equation 4.

$$\sigma_{max} = \frac{\tan^{-1}(2R)^{-1}}{3} \qquad (4)$$

Using $\sigma_{max}$ from Equation 4 yields a 0.999989 probability of a plausible location for the various ranges. A simulation was used to verify these ideas. The simulation indicated a bias of 7% at ranges calculated from Equation 3. Bias is discussed in Section 4.

It is clear from Figure 9 that for a fixed $\sigma$ (LOB error), making $\theta$ smaller increases the probability of an implausible location. Figures 10 and 11 show $\theta$ as a function of range. By forming the ratio of $\theta$, which is a function of range, and $\sigma$, a standard descriptor of system plausibility is achieved. This ratio can be thought of as indicating the stability of a particular LOB system-target geometry. This ratio will be referred to as the LOB system stability ratio, or more simply, the stability ratio. Note that for a fixed error, the stability ratio decreases as the range to the target increases. This is displayed graphically in Figures 12 and 13.

For stationary targets and sensors when the safe range has been exceeded, there are several techniques that can be used to estimate the target location. One method would use the location associated with the median range as the estimate of the target location. Using this method, all locations that are reported behind the sensors should be considered to be at infinite range. A second method involves searching for

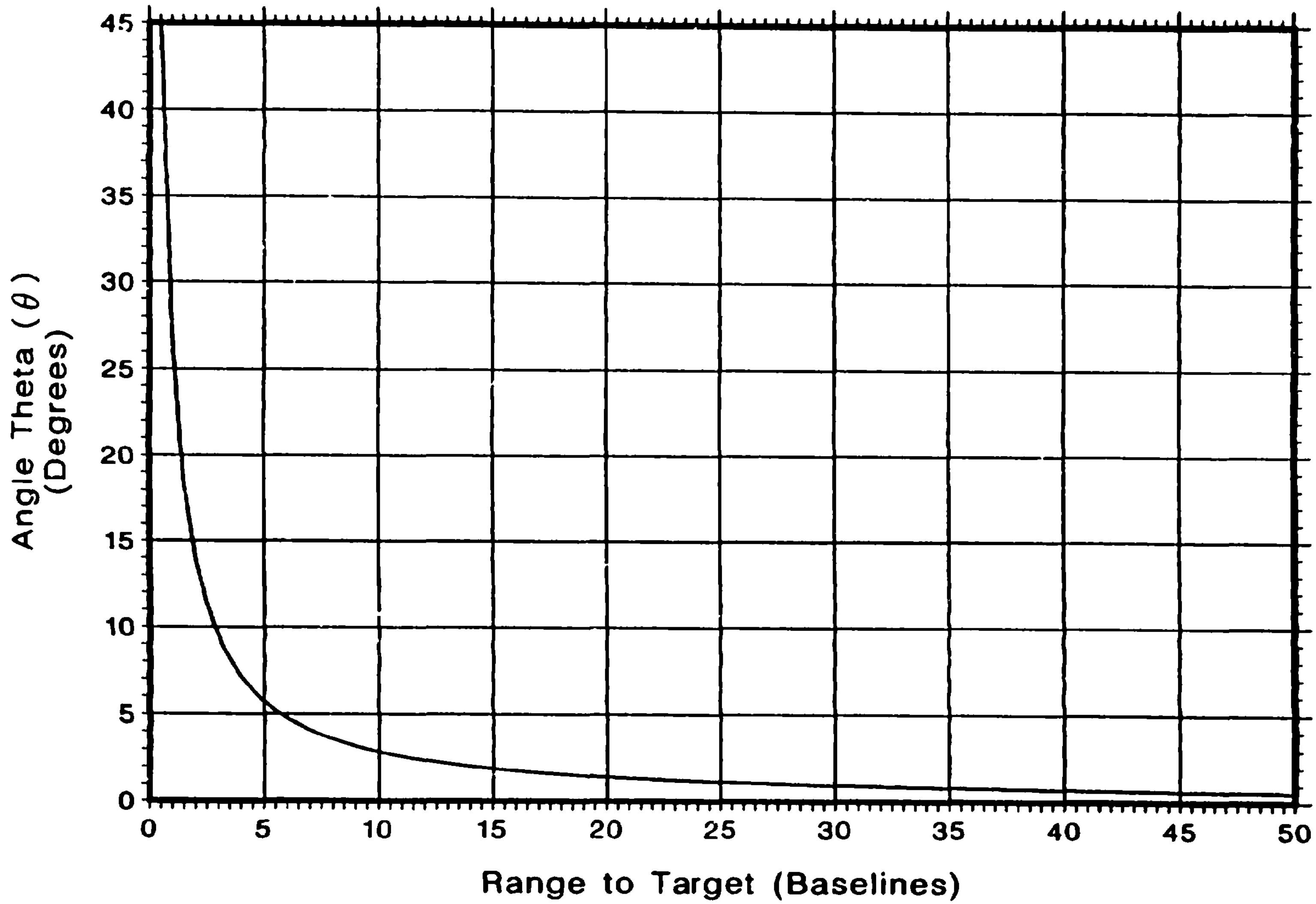


Figure 10. Theta ($\theta$) vs range-to-target for ranges to 50 baselines, target on-boresight.

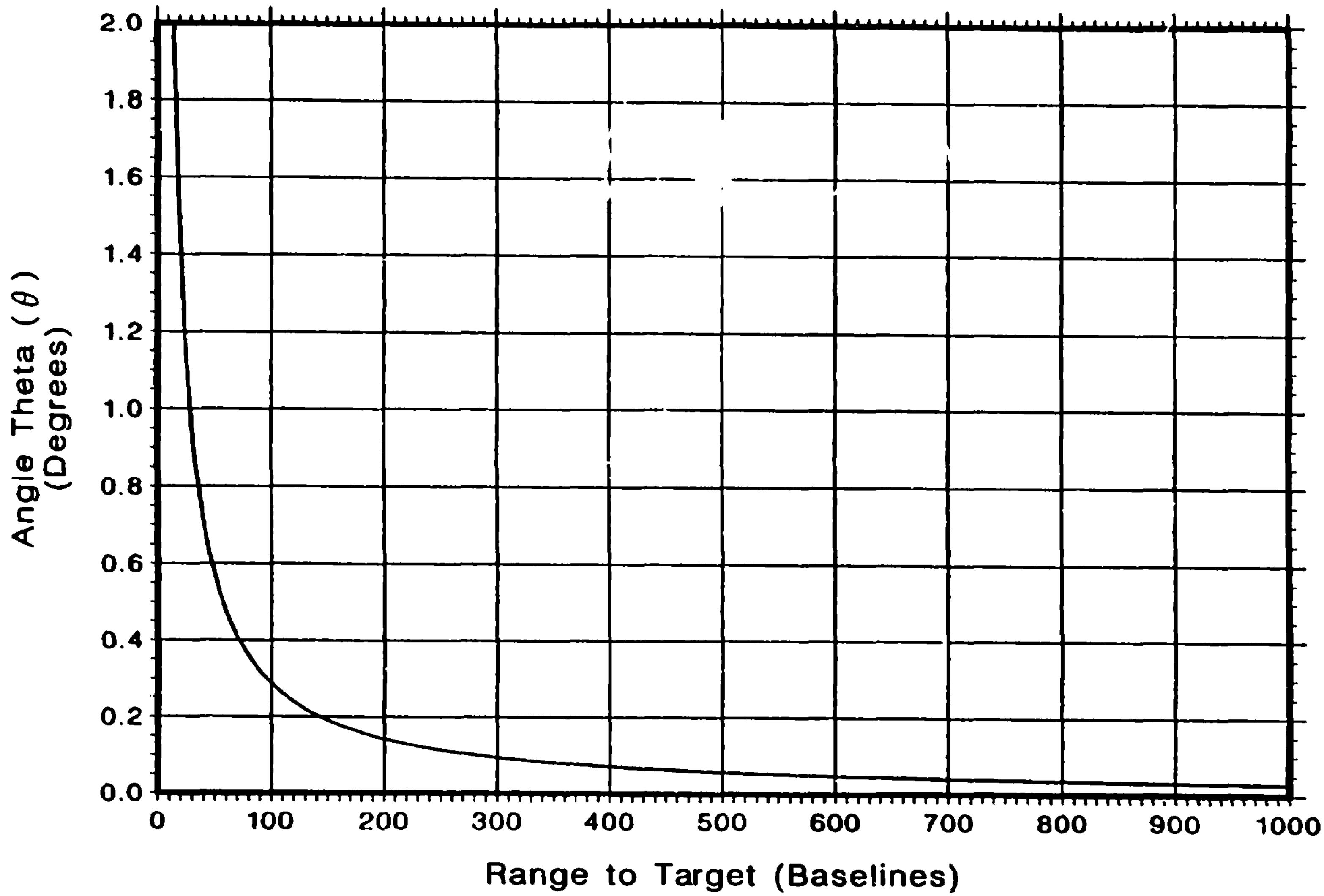


Figure 11. Theta ($\theta$) vs range-to-target for ranges to 1,000 baselines, target on-boresight.

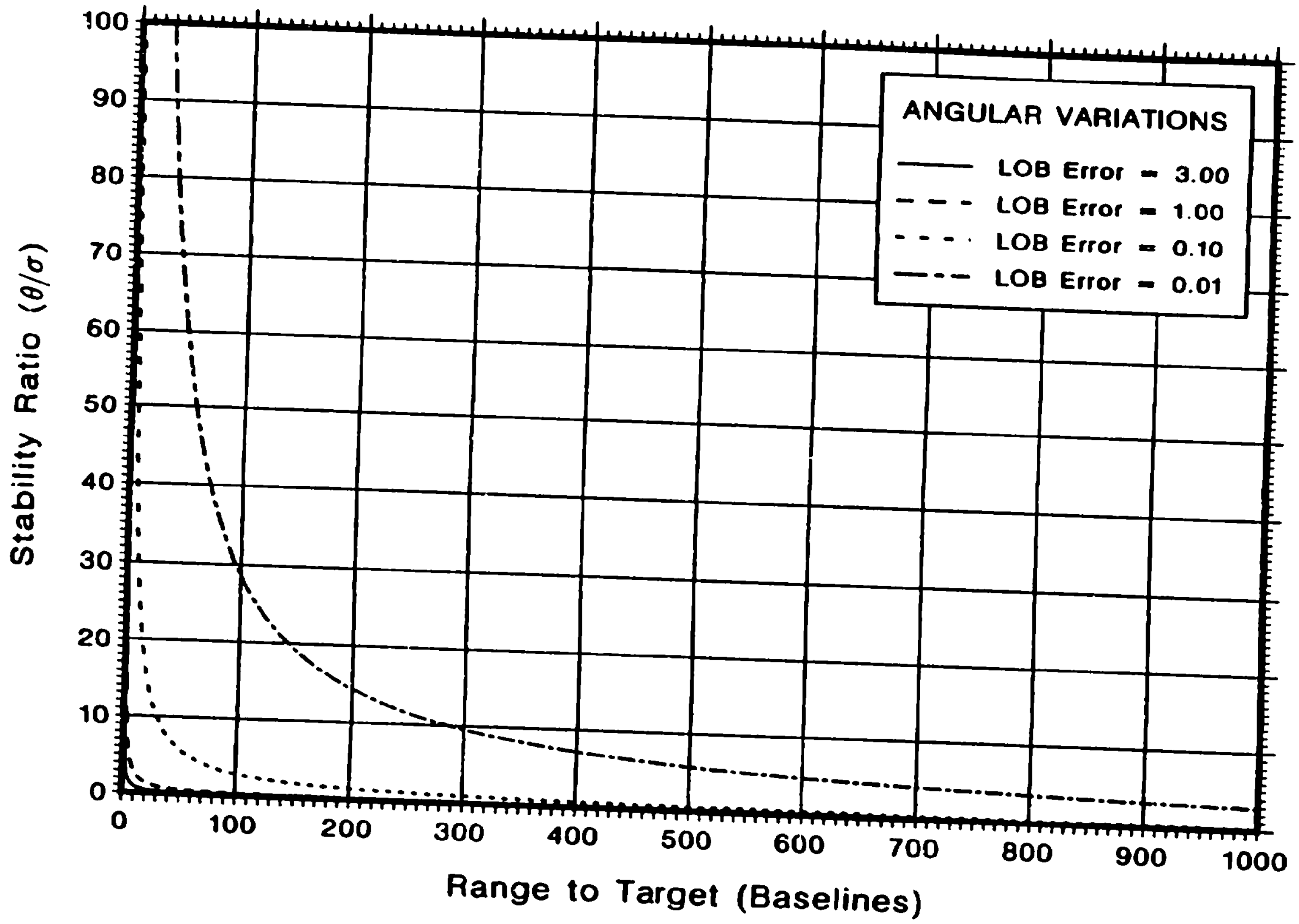


Figure 12. Stability ratio (θ/σ) vs range-to-target for ranges to 1,000 baselines, target on-boresight.

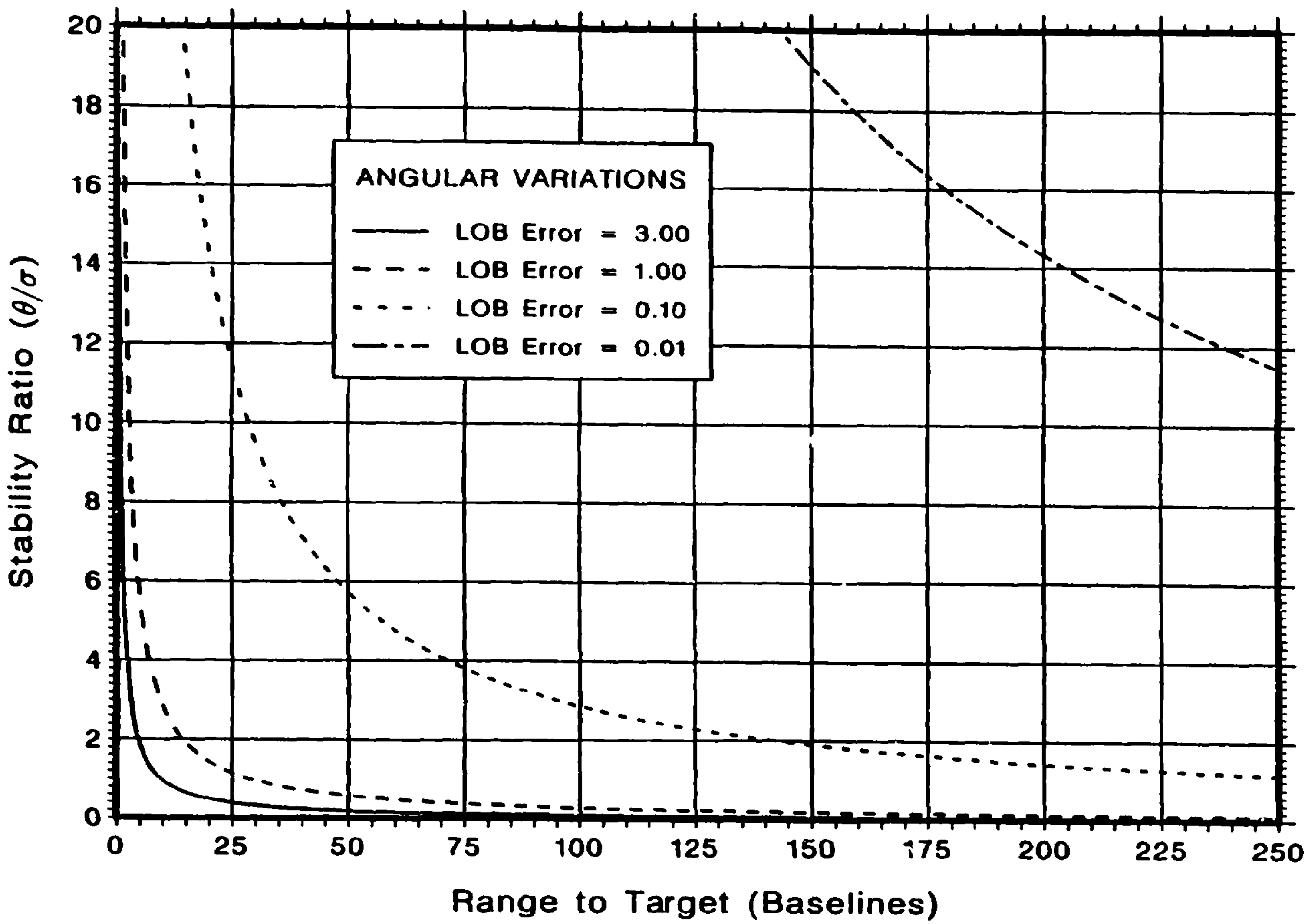


Figure 13. Stability ratio ($\theta/\sigma$) vs range-to-target for ranges to 250 baselines, target on-boresight.

the point that minimizes the squared perpendicular distances between each LOB and the point. This technique requires iterative processing. The major problem associated with these methods is the need to store all past measurements. Another method would be to average the LOB measurements at each sensor, and then use these estimates to calculate the estimate of the target's location. If there are many targets, it may be difficult to decide which target to associate with a given LOB measurement. To solve this data association problem, it may be necessary to calculate a location from a given pair of LOB measurements in order to select a potential target to associate with the measurement. This method will reduce the angular error to $\sigma/\sqrt{N}$, where N is the number of independent measurements of the angle.

## 4. BIAS

The long tails of the probability distribution in the direction of greater range cause a bias when the mean is used as the estimator of the xy position (see Figure 3). This bias increases the estimate so that it moves further from the true location in range. For cases of poor geometry, one value can dominate the estimate. As an example, suppose one of the cuts was at negative infinity. Although points of exceptional influence are real possibilities in terms of the error distributions (they are typically outside of the sensor's field of view due to range or directional constraints), some systems are designed to ignore such points. The simulation was used to find the bias for various values of the stability ratio. Recall this ratio becomes smaller as the range is extended and gives an indication of the stability of the system. Ratios greater than 3, P(implausible point)<0.000011, are considered acceptable geometries. Ten thousand points were generated for each cell within the table.

Table 1. Bias as a Percent of the True Value

| | Stability Ratio ($\theta/\sigma$) | | | | |
|---|---|---|---|---|---|
| $\sigma$ | 2.5 | 3 | 4 | 5 | 6 |
| 3 | 10.5 | 6.6 | 3.4 | 2.1 | 1.3 |
| 1 | 12.6 | 7.2 | 3.6 | 2.3 | 1.7 |
| 0.1 | 14.0 | 6.7 | 3.2 | 2.2 | 1.5 |
| 0.01 | 12.0 | 7.3 | 3.5 | 2.1 | 1.3 |

The results indicate that there is an approximate bias of 7% when the ratio is 3, 3.4% if the ratio is 4, and 2.2% if the ratio is 5. For ratios less than 2.5, the bias goes up dramatically. The estimates for systems with poor geometries can be dominated by a single extreme value.

These results were also confirmed for targets located 45° off the system boresight. For targets not on the boresight, an effective baseline needs to be calculated before the range to the target is found. If the target is $\beta°$ off-boresight, then the effective baseline is the $\cos(\beta)$ baseline units.

## 5. ERROR DISTRIBUTIONS

In this section, the agreement between the closed-form model and the simulation results was investigated. The agreement will be evaluated probabilistically at various ratios of $\theta$ to $\sigma$. The first task was to find the region within which the cuts on target follow a bivariate normal distribution.

This paragraph describes the procedure used to test for the normality of a set of cuts. For a specific geometry and LOB error ($\sigma$), a set of 1,000 points was generated. Using the sample statistics, these points were standardized; that is, they were translated and rescaled in order to have zero mean and unit variance in each dimension. Under the assumption of circular normality, ten equally probable range bins and ten equally probable angular bins were defined. Figure 14 graphically shows the range angle bins used for this test. The shading in the figure is to emphasize that each cell has the same probability of containing an observation regardless of physical size. The expected number of observations in each cell was ten. Under the assumption of circular normality, the sum of the squared deviations from the expected value divided by the expected value should follow a chi square distribution. A probability of less than 0.05 associated with a chi square test would typically be sufficient for rejecting the circular normal assumption. A chi square test was used to evaluate the agreement between the expected and actual bin totals. Table 2 shows the typical results. Within each cell of the table is the probability of exceeding the calculated chi square value.

First notice that all the values for stability ratios of 10 are greater than or very close to 0.05. A general guideline from Table 2 would be that if the LOB measurement error, $\sigma$, is 3° or less and the stability ratio is greater than 10, the location errors will have a bivariate normal distribution. In this table, the same random number stream was used within each cell. Among the many different random number streams that were used, the one shown in this table was typical of the ones observed. The entries in the table show the effects of changing the LOB error and the target location; these effects would be masked if different random number streams were used for each cell.

| Circle Number | Probability of Being Inside Circle | Circle Radius |
|---|---|---|
| 1 | 0.10 | 0.4590436 |
| 2 | 0.20 | 0.6680472 |
| 3 | 0.30 | 0.8446004 |
| 4 | 0.40 | 1.0107677 |
| 5 | 0.50 | 1.1774100 |
| 6 | 0.60 | 1.3537287 |
| 7 | 0.70 | 1.5517557 |
| 8 | 0.80 | 1.7941226 |
| 9 | 0.90 | 2.1459660 |
| 10 | 1.00 | ∞ |

| Sample Annular Segments | Probability of Being Within Annular Segment |
|---|---|
| A | 0.01 |
| B | 0.01 |
| C | 0.01 |
| D | 0.01 |

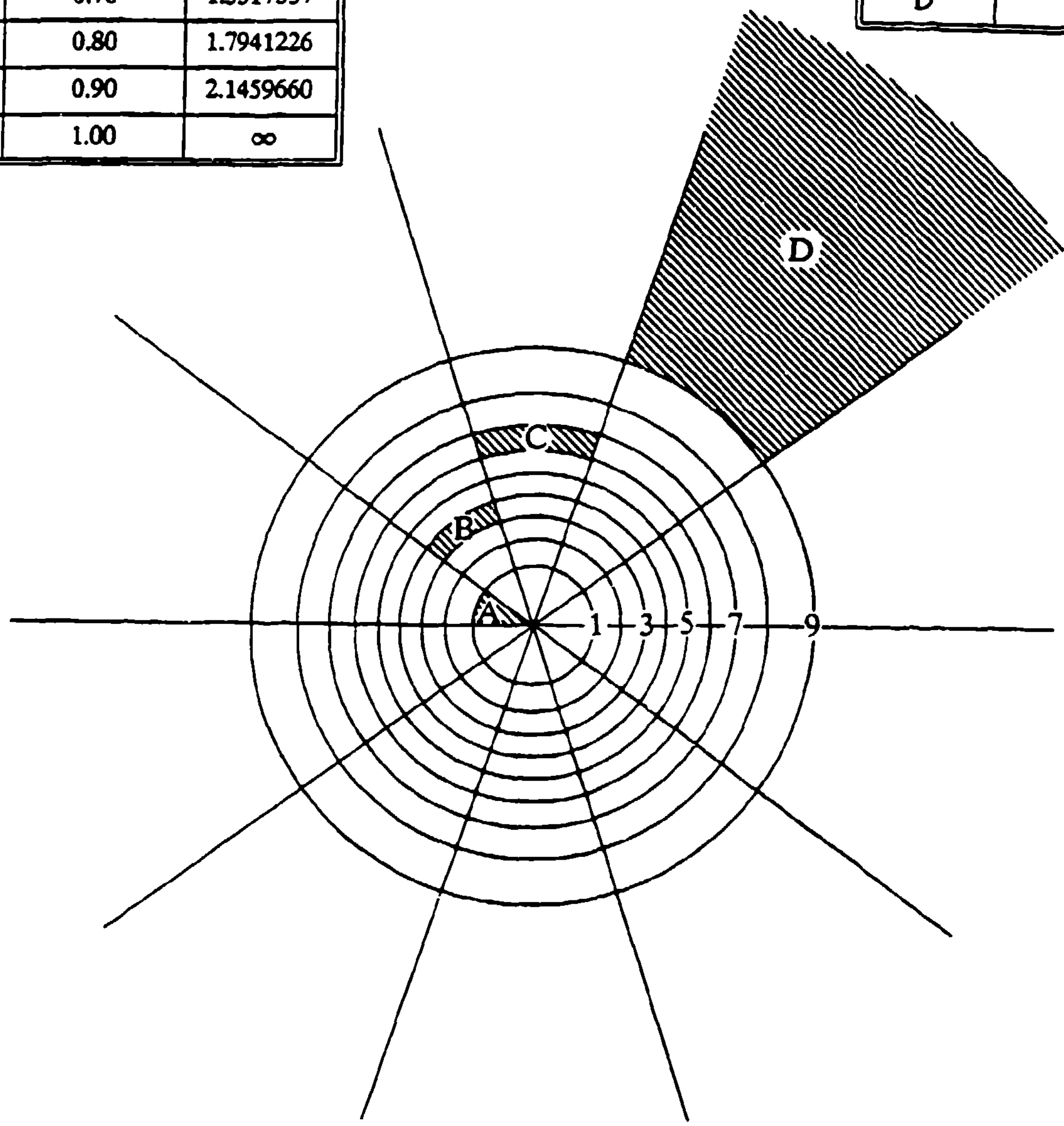


Figure 14. Equally probable range-angle bins for a circular normal distribution.

Table 2. Agreement between Simulation and Normality Assumption

| | Stability Ratio ($\theta/\sigma$) | | | |
|---|---|---|---|---|
| $\sigma$ | 6 | 8 | 10 | 12 |
| 3 | 0.0003 | 0.02 | 0.24 | 0.12 |
| 1 | 0.00003 | 0.004 | 0.05 | 0.38 |
| 0.1 | 0.00002 | 0.003 | 0.045 | 0.17 |
| 0.01 | 0.00002 | 0.003 | 0.047 | 0.17 |

For targets located 45° off-boresight, Table 2 represents equivalent systems except for one situation. The table gives a pessimistic outlook when the ratio of the distances from each sensor to the target increases beyond 1.1 or decreases below 0.91. When the target is significantly closer to one of the sensors, the agreement may be significantly better than that indicated by Table 2. This improved agreement is due to the reduced magnitude of the error associated with the closer sensor.

## 6. CLOSED-FORM MODEL COMPARISON

The next part of the investigation involved the comparison of closed-form models with the sample statistics from a simulated set of LOB locations. Both the model LOBCA, devised by Charles Alexander, and a model based on the discussion of covariance models in Thompson (1991b) were used. Despite being based on different arguments, these two closed-form models gave identical results.

The comparison of the closed-form models with the simulation was based on a test recommended by Anderson (1984). This test, based on the asymptotic distribution of the sample covariance, was used for this comparison. The assumption under the null hypothesis is that

$$\sqrt{n}\left\{\frac{\det(S)}{\det(\Sigma)}-1\right\}$$

is distributed as N(0,2p), where p is the dimension of the matrix, n is the degrees of freedom, S is the covariance of the simulation data, and $\Sigma$ is the covariance from the closed form model. In this situation, the dimension of the covariance matrix is 2, so the variance under the null hypothesis is 4. The number of samples was set to 1,000. Under these conditions, the null hypothesis was accepted at the $\alpha = 0.1$ level

for stability ratios greater than 5. Table 3 shows the probability of being larger than the calculated z value. Since a two-tailed test is being used, test values greater than 0.05 indicate that the closed-form covariance estimate does not differ from the simulation covariance. As in the previous table, the same random number seed was used for each cell. Many other runs were made with different random number seeds with similar results.

Table 3. Agreement of Closed-Form Models With a Simulation.

| | Stability Ratio ($\theta/\sigma$) | | | |
|---|---|---|---|---|
| $\sigma$ | 4 | 5 | 6 | 7 |
| 3 | 0.000036 | 0.056 | 0.400 | 0.805 |
| 1 | 0.000008 | 0.029 | 0.269 | 0.613 |
| 0.1 | 0.000007 | 0.027 | 0.255 | 0.586 |
| 0.01 | 0.000007 | 0.027 | 0.255 | 0.588 |

As in Table 2, the effect of making the measurement error smaller than 1° does not dramatically alter the agreement. Changes in the stability ratio have a considerable effect. The crossover to agreement occurs when this ratio is slightly greater than 5. For ratios less than 5, the closed-form models underestimate the diagonal components of the covariance matrix.

Table 3 is also representative of targets 45° off-boresight. This was checked using an equivalent system as described in a previous section.

When using one of these closed-form covariance models to generate the observations covariance, the Kalman filter's performance will be adversely affected if the stability ratio is less than 5. For observations in this region, two options are possible. First, any observation in this region should obtain the same large covariance. This option is based on the idea that the observations from this region can be used to initialize the filter but do not deserve much confidence. Second, the simulations could be used to determine the covariance of the observations at various ranges and off-boresight angles. The covariances in the table would be based on any gating practices such as ignoring any observations that appear behind the baseline. The table could be used to look up the proper covariance to associate with a given observation.

## 7. CONCLUSIONS

This report has discussed the various features of LOB locations based on noisy measurements. The feature used to distinguish different situations is the stability ratio, defined as the ratio of the angle formed by the range line segment and the segment connecting the target and sensor over the angular measurement error. This ratio can be used to identify several important situations. Through this ratio, the performance of a system can be predicted. For example, at ratios of 10 and greater, a bivariate normal distribution models the system adequately. If the stability ratio is greater than 5, closed-form models can be used to estimate the covariance to associate with an observation. When filtering data with stability ratios less than 5, the closed-form models will underestimate the covariance. In a Kalman filter, this can be heuristically compensated for by increasing components of the state covariance by an appropriate factor. When the ratio is less than 3, caution should be used as implausible reports become more likely. Bias varies inversely with the stability ratio. If the target and sensors are stationary, the LOB measurements can be averaged at each sensor to find the best estimate of each LOB. These averaged LOB estimates can then be used to estimate the target location.

Other estimation techniques for stationary geometries could be used. Median estimation or iterative searching estimation may be more accurate in some situations. These methods require all past data to be stored and are also computationally intensive. This is in contrast to techniques based on recursive estimators. It would be difficult to improve on the convenience of processing target locations. The recursive techniques for this method yield a current estimate with minimal storage requirements. The ideas proposed within this report allow the system user or engineer to minimize problems caused by faulty assumptions about LOB errors.

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